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Students' Mathematical Argumentation Ability Based on Mathematical Material and Semiotic Approach

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Abstract. This research aims to describe students' arguments in solving real analysis problems with a semiotic approach and describe students' arguments in solving spatial geometry problems with a semiotic approach. This study uses a qualitative method. The research subject was student teacher candidate STKIP PGRI Jombang Mathematics Education Study Program semester 4. Data collection techniques were carried out using the Proofing Problem Solving Task (TPMP) instrument, interview guide, and video recording. The research procedure included the subject being given a mathematical problem and asked to as much as possible express everything that was thought by voicing during the problem-solving process, the subject conveying the arguments that had been compiled to other fellow students in critical discussions, task-based interviews based on the results of written arguments. And verbal or subject responses. The stages of data analysis include data transcripts, data reduction; data code; checking data validity or data triangulation, reviewing data, interpreting findings, validating findings, and drawing conclusions. The results showed that the subject's mastery of mathematical material was used to build the content and quality of the argument. At the same time, the subject uses the semiotic approach to convey arguments in the form of speech, symbols, and gestures. The student's argument in solving real analysis and spatial geometry problems with a semiotic approach is that the subject conveys data, claims, warrants, backing, and rebuttal with speech, symbols, and gestures using the index finger. When the subject conveys the qualifier and rebuttal, it is dominated by speech and gesture. The subject said that the qualifier of the conclusion was certain, and there was no objection. The subject's gesture is with a firm facial expression, a head nod and a hand wave indicating no rebuttal, and using symbols as a sign of leverage.

INTRODUCTION

Argument are needed to determine, generate and support reasonable solutions when solving a problem, especially a proof problem[1]–[8]. The arguer tries to show and ensure the correct view using an argument addressed to the audience. The arguer tries to convince the audience there is something to approach and debate.

Building an argument does not only link the premise and conclusion;Toulmin's scheme is used to obtain a more complex argument analysis[9]. The Toulmin scheme consists of data, claims, warrants, backings, disclaimers, and qualifiers. Data are facts that are used to support claims. A claim is a statement that will be shown to be true. The warrant is a hypothetical statement as a bridge and justification of steps. The backing provides further evidence, namely the legal basis for the warrant. Disclaimer is the exception condition to the argument, and the qualifier expresses the level of power that the data give to the claim by the warrant.

Argumentation skills are needed when solving problems[10]. Students must have argumentation skills to justify their solutions and actions, leaving doubts and indecision in solving a problem. However, he is also more accessiblewhen choosing and can propose rational solutions. At college, students are more often faced with proving problems. Mathematical materials emphasise the ability to prove, including Algebra, Analysis, Spatial Geometry, and Calculus. The concepts of fundamental analysis and geometry of space are mathematical subjects studied at universities. Before studying the material, students must understand geometry, abstract algebra, number theory, and

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calculus before studying the material. Students have studied spatial geometry from elementary school to college. In contrast, the prerequisites for accurate analysis are studied mainly by students in higher education. In addition, the concept of fundamental analysis is more abstract than the geometry of space. Thus, research that focuses on Real Analysis and spatial geometry need to be carried out on an ongoing basis, including studying student arguments. In developing students' mathematical argumentation skills, educators need to pay attention to prerequisite material [11], [12]

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Semiotics in mathematical arguments can be interpreted as the process of producing signs and symbols as part of a code system used to communicate information and show and ensure correct information. Semiotics include visual, verbal, tactile, and olfactory signs. Steinbring states that mathematical signs or symbols are tools for coding, describing, communicating, and generalising mathematical knowledge [13]. Therefore, semiotics is very appropriate to apply in mathematics [14] because understanding abstract mathematical operations and contexts are impossible without certain activities [15].

[14] and [16] refer to several studies that refer to semiotics. [14] describes a semiotic system consisting of signs, sign production rules, and the underlying meaning structure in studying numbers, counting, and computation. [16] describes the integration of speech and gesture. While research on mathematical arguments, including[17]developed arguments and evidence for high school students under the guidance of a teacher, [18]described the argumentative process into four classifications, namely working exclusively through algebraic manipulation, brief exploration with examples and switching. In addition, to algebraic evidence, expanded exploration with examples that lead to reasoning about conjectures, and unfocused exploration with examples, [19]develop students' argumentation skills through infusion learning models. It appears that no previous research has discussed semiotics in argumentation.

Research on the semiotic approach in this argument is essential because semiotics have an important role in justification[20]. Argumentation is one of the justification processes used to show and ensure the correct view of arguments directed at oneself or others [12]. Through argument, someone tries to convince himself or others so that there is something to approach and argue to reach an agreement in justification.Thus, the objectives to be achieved in this study are: (1) to describe student arguments in solving real analysis problems with a semiotic approach; (2) to describe students' arguments in solving spatial geometry problems with a semiotic approach, (3) characterisation of students' argument skills based on mathematical material with a semiotic approach.

LITERATURE REVIEW

Math Material

One of the abstract and complex courses is in Real Analysis. In this course, students study Real Number Sequences, which contain definitions, theorems, and their proof use [21], [22]. Although abstract and highly complex, this material must be mastered by students to understand further concepts in Real Analysis, such as the number of infinite series [23]–[25].

The prerequisite material for the Real Analysis course includes abstract algebra, number theory, and calculus. These prerequisite materials are mainly studied at the university and high school levels. Meanwhile, students also have difficulty in learning this prerequisite material [26], [27]. As a result, students do not understand and develop the concept of Real Analysis.

In addition to real analysis, students also study spatial geometry. The geometry of space or three-dimensional geometry is a part of the geometry that discusses spatial or three-dimensional shapes [28], [29]. For example, building space can be defined as a shape not entirely located on the plane because it contains three elements: length, width, and height [29]. Three-dimensional geometry discusses objects that are abstract [28]. These objects are points, lines, planes, cubes, blocks, spheres, and so on, all of which are objects obtained through an abstraction process based on concrete objects found in everyday life [30], [31]. Thus, it is highly expected that educators and teachers in developing students' understanding of geometric concepts should be carried out by manipulating spatial objects or real objects (concrete experience).

The prerequisite material for the spatial geometry course includes flat plane geometry. These prerequisite materials are mostly studied at the university level, high school,junior high school, and elementary school. Students' difficulties in understanding the concept of Spatial Geometry are imagining spatial concepts, making illustrations or drawing a three-dimensional shape, and solving spatial geometry problems related to proof [32]–[36].

Semiotic Approach

Semiotics is a term that Charles S. Peirce first used to designate a sign of an action or a sign of a process, in general, sign of a sign activity [37]. Semiotics study sign systems, such as words, signals, and gestures. Semiotics related to the production of signs and symbols is part of the code system used to communicate information. Semiotic systems have three purposes: to show mathematical objects, communicate, and work on mathematical objects themselves [38]. Semiotics includes anything that has a relationship with a sign, starting from the sign being sent, received, functioning, and concerning something else[39].

The types of signs in mathematics education include embodiment, gestures, and body [40]. Embodiment theory originating from cognitive linguistics generally assumes a mediator, such as a schema that connects the mind with the real world, for example, the APOS theory, three worlds of mathematics. Gestures are arm and hand movements, usually in front of the body for symbolic expression [16]. Only the movements are considered communicative acts by the speaker and listener.

[16]classifies gestures as deictic, metadhroric, iconic, and beat. A deictic gesture is a gesture that points to an existing object. Metadhroric gestures refer to gestures whose contents represent abstract ideas without physical form. The iconic gesture is defined as a gesture that emphasises a similar relationship with the spoken semantic content. Finally, the beat gesture is a simple gesture that is repeated and used for emphasis.

The basic components of semiotics consist of: signs, symbols, and cue [41]. In this case the sign always points to something real, for example, objects, events, writing, language, actions, events, and other forms of signs. A symbol is always associated with signs that have been given cultural, situational, and conditional characteristics. A symbol is a sign that means dynamic, special, subjective, figurative, and figure of speech. Examples of color symbols, symbols of objects, symbols of sound, symbols of atmosphere, symbols of tone, and symbols of imaginative visualization caused by facial makeup or typography. A cue is a thing or condition given by the subject to the object. In this situation, the subject always does something to inform the object that is signaled at that time.

There are three semiotic components, namely gestures, words and symbols [42]. Gesture is the movement of human body parts such as eyes, shoulders, hands and feet when communicating with other people. Gesture is naturally a reflex movement as a form of strengthening the ongoing communication message. Example: shrugging as a sign of not understanding what is being said. Word is an expression of expression in the form of words or sentences made by humans. Symbols are expressions of something in the form of signs, pictures or letterss

RESEARCH METHOD

The research subject is student teacher candidate STKIP PGRI Jombang Mathematics Education Study Program semester 4. The reason for choosing students in semester 4 is that students are studying the concept of spatial structure and real analysis. Students have also studied the prerequisite materials for spatial construction and real analysis: flat plane geometry, abstract algebra, calculus, and number theory. Students have also attended lectures using the Infusion Learning model. The learning model aims to develop students' argumentation skills based on the Toulmin scheme [12], [19] namely they have the provision ability to convey arguments by solving the problem of proof given.

Research procedures include: first, the subject was given a mathematical problem and asked to express all that was thought as much as possible by voicing (think aloud) during the problem-solving process. So we get the argument not in dialogue. In the second stage, the subject conveys the arguments compiled to fellow students in a critical discussion. Fellow students can respond byaccepting or supporting arguments to create a situation where the subject tries to convince his partner through an approach and debate. The process in the second stage obtained data arguments in the dialogue. In the third stage, conducted a task-based interview based on the results of written and oral arguments or the subject's response.

The research instruments include Proofing Problem Solving Tasks (TPMP), interview guidelines, and video recording. TPMP and video recording are used to describe the subject argument. TPMP contains mathematical problems that contain the concept of spatial structure and real analysis. The subject solved the following questions:

Real Analysis Prove that "if a, b are members of $\mathbb{R}$, then $a + x = b$ has a single solution $x = (-a) + b$" !
Space Geometry Prove that the area of the section of the sphere $=2\pi rt$,where r is the sphere's radius and t is the height of the section!

FIGURE 1. Proof Problem Solving Task (TPMP)

The data obtained from the interview process and think-aloud were transcribed and analysed. Researchers followed three stages of qualitative data analysis activities from [43] and six stages of qualitative data analysis and interpretation based on Creswell's theory[44]. The stages of data analysis include data transcripts, data reduction; data code; checking data validity or data triangulation, reviewing data, interpreting findings, validating findings, and drawing conclusions. The validity of the data used in this study is time triangulation, namely by giving a proof problem solving test and interview at least twice at different times. If the test results produce different data, then it is done repeatedly so that data that is stable or credible is found.

RESULTS AND DISCUSSION

The researchers involved 80 prospective mathematics education teacher students at a university in Jombang, East Java, Indonesia. The participants consisted of 38 men and 42 women. This research was conducted from April 5 to June 10, 2022. The researcher chose 1 participant as the subject, namely the participant who conveyed an argument according to the components of the Toulmin schema and showed expressions so that the semiotic approach could be observed.

Students' Arguments in Solving Real Analytical Problems with A Semiotic Approach

The subject conveys the data by saying that "a and b are elements of real numbers." The symbol used by the subject is "element ($\in$)," which means "member". The subject's gesture is the subject using the index finger, which leads to the sentence "a and b elements of real numbers." There is a match between the speech, symbols, and gestures of the subject when conveying data.

The subject conveys the claim by saying that "$a + x = b$ has a single solution, namely $x = (-a) + b$". The subject uses the sign "if...then... ($\rightarrow$)"; the sign is interpreted as the word "single solution." The subject's gestures appear when using a ballpoint pen to point at $x = (-a) + b$, and the eyes are not directed towards the sentence so that there is no match between symbols, speech, and gestures when the subject makes a claim. The following is a photo of the subject, when submitting a claim

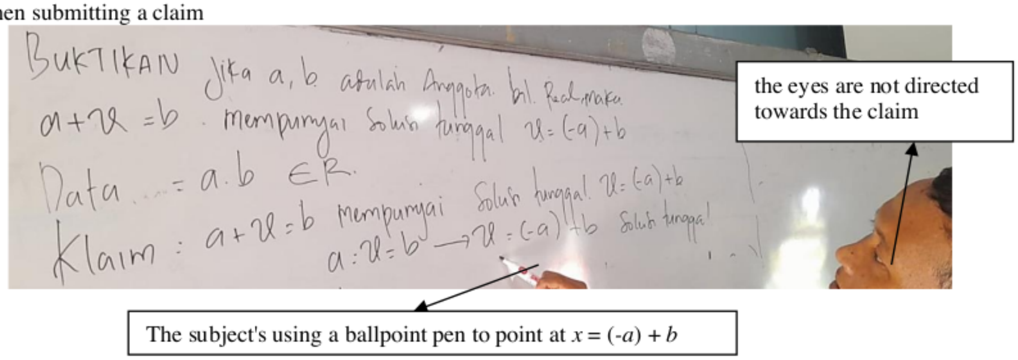


FIGURE 2. The subject conveys the claim

The subject conveys the warrant with words as written; namely, the subject proves that the solution of $a + x = b$ is $x = -a + b$, then shows that $x = -a + b$ is a single solution. The symbol used by the subject is to write a symbol of a mathematical formula that indicates the result of an algebraic operation. The subject guide includes the index finger indicating each step of problem-solving. There is a match between the speech, symbol, and gesture of the subject when conveying the warrant. However, the subject only conveys 1 warrant.

The subject conveys the backing with speech, namely algebraic operations, addition inverses, and identity elements. The symbol used by the subject is "brackets ()" which is defined as "the basis of the step". Subject gestures include using a ballpoint pen to indicate backing and underlining. There is a match between the speech, symbol, and gesture of the subject when conveying the backing. Here's a photo of the subject when delivering the backing in Figure 3.

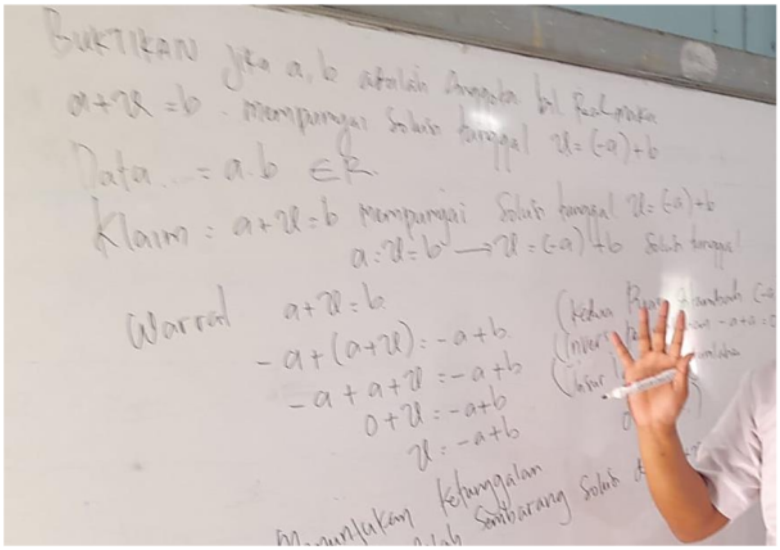
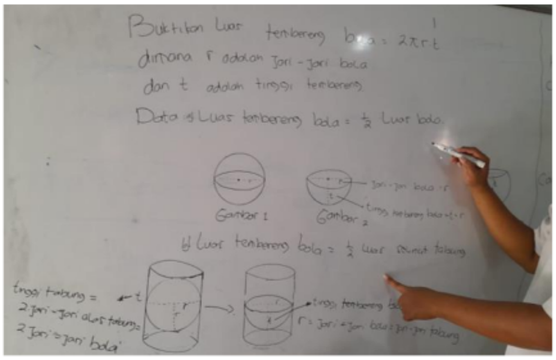


FIGURE 3. The subject conveys the qualifier and rebuttal

The subject conveys the qualifier and rebuttal simultaneously. The subject said that the qualifier of the conclusion was cerain, and there was no objection. The subject's gesture is with a firm facial expression, a head nod, and a hand wave indicating no objection as Figure 4. The symbol used is a sign of verification (checking) that is rare in solving the problem of proving that there is no refutation and it is correct.

Student Arguments in Solving Spatial Geometry Problems with A Semiotic Approach

The subject conveys the data by speech covering the area of the ball section = $\frac{1}{2}$ the area of the tube blanket, the area of the ball section = $\frac{1}{2}$ the area of the ball. The subject reveals the known data on the question, and the data is associated with the knowledge possessed by the subject. The symbol used by the subject is to sketch an image representing the area of the sphere. The subject's gesture is to show a sketch of the image of the section with a marker. There is a match between speech, symbols, and gestures of the subject when conveying data. The following is a photo of the subject when submitting the data as Figure 4.



Direct Translation:

Prove the breadth of the ball.
Where r is the radius of the ball
And t is the height of tembereng
Data: a) ball area = $\frac{1}{2}$ ball area
b) spherical span area = $\frac{1}{2}$ area of blanket tube

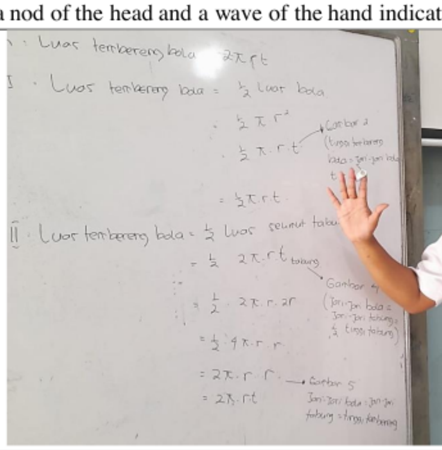
FIGURE 4. The subject shows the data by using a picture of a the ball section

The subject conveys the claim by saying, "area of the spherical section = 2πrt". The subject writes the symbol of a mathematical formula that represents equal to, the ball's radius and the segment's height. The subject's gestures appear when using a ballpoint pen to point at 2πrt. When the subject makes a claim, there is a match between symbols, speech and gestures.

the subject conveys the warrant by saying as written, namely the subject proving the area of the spherical segment = 2πrt through the area of the tube blanket, then proving the area of the spherical segment = 2πrt through the area of the half sphere. The subject guster includes the index finger indicating each step of problem solving. The symbol used by the subject is "equal to =" indicating algebraic equality. There is a match between the speech, symbol, and gesture of the subject when conveying the warrant. The subject conveys 2 warrants in his argument. The following is a photo of the subject when submitting the warrant as Figure 5.

The subject conveys backing with speech, namely algebraic operations, the similarity of the height of the tube blanket with the radius of the tube, the similarity of the radius of the ball with the height of the segments. Subject gestures include the use of a ballpoint pen to indicate the backing. The symbol used is “arrow ” which shows the meaning of equal value. There is a match between the speech, symbols, and gestures of the subject when conveying the backing.

The subject conveys the qualifier and rebuttal which is dominated by speech and gesture. The subject said that the qualifier of the conclusion was certain and there was no objection. The subject's gesture is with a firm facial expression, and a nod of the head and a wave of the hand indicating no objection as Figure 5.



Direct Translation:

I. Ball tembereng area = $2\pi rt$
II. Ball tembereng area = $\frac{1}{2}$ ball area
III. Ball tembereng area = $\frac{1}{2}$ spacious blanket tube

FIGURE 5. The subject conveys the qualifier and rebuttal

Characterisation Of Students' Argument Skills Based On Mathematical Material With A Semiotic Approach

Based on the results of the description of mathematical arguments in proving space geometry problems and real analysis, it is obtained characterisation of students' argument skills based on mathematical material with a semiotic approachas shown Table 1.

TABLE 1. Characterisation of students' argument skills based on mathematical material with a semiotic approach

Argument Component	Semiotic Components in solving real analysis proof problems			Components of Semiotics in solving the problem of proving the geometry of space		
	Utterance	Symbol	Gesture	Utterance	Symbol	Gesture
Data	convey data based on what is known in the question	Write symbols that indicate members (∈) and equality (=)	Show data with finger expression	convey data based on what is known in the problem and relate the data to the knowledge possessed	sketch a picture that represents a group of balls	Show data with finger expression
Claim	submit a claim based on what is known to the question in spoken and written words	Write a symbol that signifies "if…then…. (→)"	Showing claims with finger expressions but eyes that don't lead to claims	submit a claim by saying that "area of the spherical segment = 2πrt," which is in accordance with the writing	write a mathematical formula symbol that represents the radius of the ball, and the height of the segments	Showing claim with finger expression
Warrant	delivery of speech as written, namely proving that the solution of $a + x = b$ is $x = -a + b$, then showing that $x = -a + b$ is a single solution	write a mathematical formula symbol that indicates the result of an algebraic operation	an index finger that shows each step of problem-solving	Convey warrants with words as written. More than 1 warrant is submitted due to the results of linking between spatial shapes	Writing an "equal to =" sign indicates algebraic equality	Using the index finger to show each step of the problem solving
Backing	pronounce algebraic operations, addition inverses and identity elements in addition as written	Using the symbol "open and close parenthesis ()" which is interpreted as the basis for solving the proof problem	use a pen to show the backing and underline	convey backing with speech and writing, namely algebraic operations, the similarity of the height of the tube blanket with the radius of the circle, the similarity of the radius of the sphere with the height of the segments	Using the symbol "arrow," which shows the meaning of the same value	using a ballpoint pen to show the backing

TABLE 1. Characterisation of students' argument skills based on mathematical material with a semiotic approach (Continued)

Argument Component	Semiotic Components in solving real analysis proof problems			Components of Semiotics in solving the problem of proving the geometry of space		
	Utterance	Symbol	Gesture	Utterance	Symbol	Gesture
Qualifier and Rebuttal	Submission of qualifier and rebuttal is based on the warrant used	a firm facial expression, and a nod of the head and a wave of the hand indicating no rebuttal based on the warrant used	Write down the verification marks (checks) that are rare in solving the problem, the proof is correct and there are no objections	Submission of qualifier and rebuttal is based on the warrant used	a firm facial expression, and a nod of the head and a wave of the hand indicating no rebuttal based on the warrant used	Write down the verification marks (checks) that are rare in solving the problem, the proof is correct and there are no objections

There are differences in the ability of mathematical arguments to solve real analysis problems and spatial geometry due to the character of the material. The subject has studied geometry material since elementary school [45] so that the subject can connect the interrelationships between shapes in geometry. In contrast, the subject began to study real analysis while in college.

The character of real analysis material is an abstract and complex concept. So that there is a discrepancy between the gesture and the subject's speech when determining the claim. This shows the subject's doubts in determining the claim. In addition, the subject conveys 1 warrant as a guarantee that the statement is true.The character of space geometry material is the relationship between shapes, so the subject uses this relationship in delivering 2 warrants.

Semiotic components in students' mathematical arguments include speech, symbols and gestures. The subject expresses the utterance in the argumentation process in the form of words or sentences both orally and writing. For example, the subject states the similarity between equation 1, equation 2, and equation 3 in the solution step by saying, "will be equal to both sides plus -*a*." While in spatial geometry, the subject stated the relationship between the shapes by saying "the volume of the ball is equal to two-thirds of the volume of the tube when the diameter of the ball is the same as the diameter of the tube and the height of the tube".

Based on the study's results, it appears that the subject uses mathematical symbols to deliver arguments using the Toulmin schema component. Mathematical symbols are tools for coding, describing, and communicating mathematical knowledge [13]. Symbols that appear in students' mathematical arguments when solving real analyst proof problems are in the form of signs such as "=" "→" and "()". While the symbols used by the subject when solving the problem of proving the geometry of space are in the form of letter marks, for example "*r*" "*t*" "*v*" and sketch images. The subject uses familiar symbols whose meaning has been understood by the subject and the subject's friends as the audience.

Gestures in student arguments when solving proof problems are in the form of finger expressions, facial expressions, or eye gaze. The process of determining data, claims, warrants, and backing can be observed from the expression of the fingers or mimicking shown by the subject. The expression of the subject's finger movements changes every step of solving the proof problem. This type of gesture is deictic, meaning it points to an existing object [16]. This gesture is a natural gesture, which is a reflex movement as a form of strengthening the ongoing communication message [42].

Meanwhile, when determining the qualifier and rebuttal, the subject showed more assertive facial expressions, sharp eyes, nodded his head, and waved his hands as a sign that his argument was true and trustworthy, and there was no rebuttal. In this case, a beat gesture is a simple gesture that is repeated and used as an emphasis [16]. Gesture representation is an action or movement that describes an abstract and concrete idea, entity, or event that is conveyed in words [46]. Gestures emphasise more effectively than words [47]. Gestures have proven helpful in supporting learning in various content, including real analysis and spatial geometry.

CONCLUSION

The student's argument in solving real analysis problems with a semiotic approach is that the subject conveys data, claims, warrants, backing, and rebuttal with speech, symbols, and gestures using the index finger. The symbol used by the subject when conveying data is "element ($\in$)" which means "member". The subject conveys data based on what is known in the question. When submitting a claim, the subject uses the sign "if...then... ($\rightarrow$)", the sign is interpreted as the word "single solution", and the subject's gesture uses a ballpoint pen to point at $x = -a + b$ and the eyes do not point towards the claim so that there is no correspondence between symbols, speech nd gestures when the subject makes a claim. When submitting a warrant, the subject uses a mathematical formula symbol that indicates the result of an algebraic operation, the subject's guster includes the index finger indicating each step of solving the problem, but the subject only conveys 1 warrant. When conveying the backing, the symbol used by the subject is "brackets ()" which is interpreted as "the basis of the step" the subject's gesture includes the use of a ballpoint pen in showing the backing and underlining. The subject conveys the qualifier and rebuttal simultaneously, the subject's gesture is with a firm facial expression, and a nod of the head and a wave of the hand indicates there is no rebuttal, the symbol used is a sign of verification (checking) namely "√"

Students' arguments in solving spatial geometry problems with a semiotic approach are the subject conveys data, claims, warrants, backing, and rebuttal with speech, symbols, and gestures. The subject reveals the data that is known in the problem and the data is associated with the knowledge possessed by the subject, the symbol used by the subject is to sketch an image that represents the area of the sphere, the subject's gesture is to show a sketch of the group image with a marker. When making a claim, using a mathematical formula symbol representing "equal to, the radius of the ball, and the height of the segment", the subject's gesture appears when using a ballpoint pen to point. When conveying a warrant, the subject's gestures includes the index finger used to indicate each step of solving the problem, and the symbol used by the subject is "equal to =" indicating algebraic similarity. When conveying the backing, the subject's gesture includes using a ballpoint pen to indicate, the symbol used is the "arrow," which indicates the meaning of the similarity of values. When the subject conveys the qualifier and rebuttal, it is dominated by speech and gesture. The subject said that the qualifier of the conclusion was certain, and there was no objection. The subject's gesture is with a firm facial expression, a head nod, and a hand wave indicating no rebuttal.

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